Domain Wall Model in the Galactic Bose-Einstein Condensate Halo

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We assume that the galactic dark matter halo, considered composed of an axionlike particles Bose-Einstein condensate [1], can present topological defects, namely domain walls, arising as the dark soliton solution for the Gross-Pitaevskii equation in a self-graviting potential. We investigate the influence that such substructures would have in the gravitational interactions within a galaxy. We find that, for the simple domain wall model proposed, the effects are too small to be identified, either by means of a local measurement of the gradient of the gravitational field or by analysing galaxy rotation curves. In the first case, the gradient of the gravitational field in the vicinity of the domain wall would be 10^{-31} $(m/s^2)/m$. In the second case, the ratio of the tangential velocity correction of a star due to the presence of the domain wall to the velocity in the spherical symmetric case would be 10^{-8}.

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I. INTRODUCTION

The existence of a mysterious kind of matter, rather different from the usual barionic matter, presents itself as a great challenge for modern Physics. This so-called dark matter corresponds to almost 23% of the energy density of the Universe [2] and can amount to approximately 90% of the total mass in galaxies.

Recently, it has been proposed that this type of matter can be composed of some kind of weakly interacting bosons [3]. When these bosons are spinless they can be identified with axions, hypothetical particles proposed in the context of Peccei-Quinn models [4]. On the other hand, in the case of sub-eV spin-1 particles, they are called hidden bosons or hidden photons [5, 6]. Axions and hidden bosons form a class of particles known as WISPs (Weakly Interacting Slim Particles), due to their diminute masses.

In the last few years, the possibility that the dark matter content of galaxies is in the form of a self-graviting Bose-Einstein condensate (BEC) has been considered. Using this approach, the authors in [7] were able to relate the mass and the scattering length of an axionlike particle with the radius of the galactic dark matter halo. By proposing a new density profile based in the BEC features they could construct rotation curves that fit well a sample of galaxies.

Using the same initial hypothesis, and extending it to spin-1 particles, the authors in [1] showed that the mass of the WISP's is constrained by galaxy radii data to the range $10^{-6} - 10^{-4}\ eV$.

The next natural step in the identification of the dark matter halo with a BEC is to study the possible presence of substructures. BEC's can present a number of different substructures, called topological defects, such as vortexes, domain walls, monopoles and textures.

The existence of these substructures is verified in laboratory experiments, for ultra-cold alkali atom gases trapped in a magnetic optical potential, and their features are well studied under these circumstances [8–10]. They are observed as a small region (much smaller than the size of the condensate) of null mass density in the gas. Mathematically, these defects have origin in zeros of the condensate wave-function, stressing the quantum nature of these phenomena in the gas.

Topological defects have also been studied in the framework of cosmology and gravitation [11], in which they have notable differences from the condensed matter ones. For instance, they can be massive and carry a large amount of energy. Recently, the authors in [12] have suggested an experiment to detect a massive axionic domain wall via magnetic interaction in the context of field theory.

To the authors knowledge, topological defects for a self-graviting BEC have never been proposed

before. Our goal in the present paper is to explore the possibility that the galactic BEC is endowed with a domain wall, a finite region in space where the density vanishes. We are interested in the effect that such a structure can have on the galactic dynamics (specially on rotation curves), and if it is possible to detect a local domain wall by means of gravitation interaction effects. We restrict ourselves to the axionlike particle case.

This paper is organized as follows. In section II we perform the derivation of the density functions for the halo endowed with a domain wall, and estimate its width. In section III, we give an estimate of the gradient of the halo gravitational field in the vicinity of a domain wall. In section IV we derive a correction term for the rotation curves of stars in spiral galaxies taking into account the influence of a domain wall perpendicular to the galactic disk plane (as depicted in figure 1). Section V shows our final remarks.

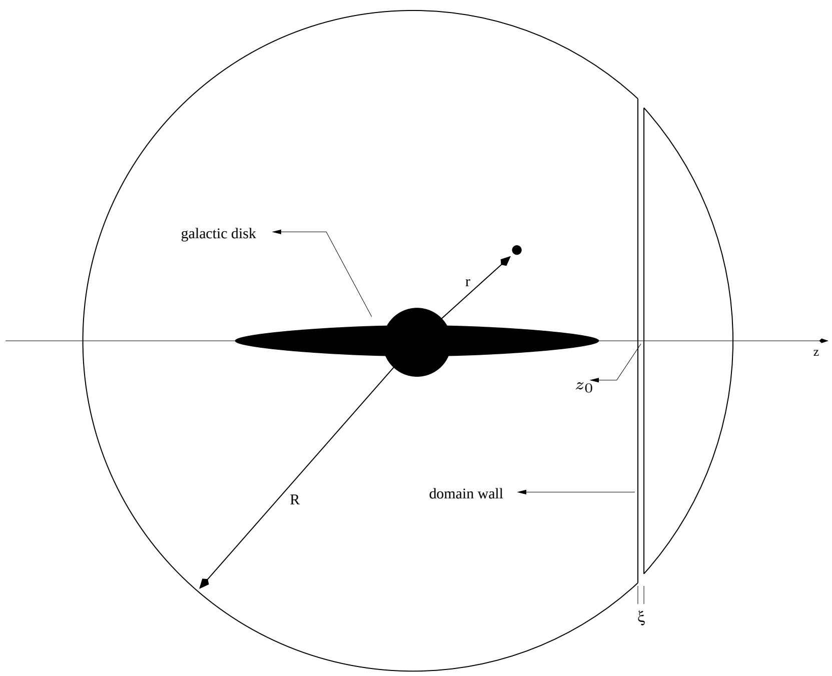

FIG. 1. Schematic view of the coordinate system used in this paper. R is the galaxy halo radius. The domain wall of width ξ is in a position z_0, and is perpendicular to the galactic plane, chosen to be the location of the z axis.

II. DARK SOLITONS

The zero-temperature mean field energy of a weakly interacting BEC confined in a self-graviting potential, V, is given by [1]

$$E = \int d^3\mathbf{r} \left[\psi^* \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi\right] + \frac{4\pi\hbar^2 a}{m}|\psi|^4 \tag{1}$$

where m is the mass of the particle composing the condensate, a is the s-wave scattering length and $\psi(\vec{r})$ is the condensate wave function, satisfying $\int d^3\mathbf{r}|\psi|^2 = N$ with N being the total number of particles.

The mean field dynamics of the system is described by the Gross-Pitaevskii (GP) equation

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + g|\psi(\mathbf{r},t)|^2\right)\psi(\mathbf{r},t)\ , \tag{2}$$

where $g = 4\pi\hbar^2 a/m$.

Some of the solutions of equation (2) may be quantized vortexes or dark solitons. These functions are topological defects in scalar BECs, in which the density vanishes due to the topological constraint on the phase of the wave function.

In order to investigate the topological defect that corresponds to the domain wall that can appear perpendicularly to the z direction, we coupled the topological defect with the ground state of the galactic condensate in the Thomas-Fermi approximation [7] in the form

$$\psi(\mathbf{r},t) \equiv \psi_{TF}(x,y,z)\phi(z,t)\ , \tag{3}$$

where $\psi_{TF}(x,y,z) \equiv \psi_{TF}(r)$ is the Thomas-Fermi solution for the GP equation (with $r^2 = x^2 + y^2 + z^2$),

$$\psi_{TF}(r) = \begin{cases} \sqrt{\rho_0 \frac{\sin kr}{kr}} \quad \text{for} \quad r \leq R \\ 0 \quad \text{for} \quad r > R \end{cases} \tag{4}$$

with $k = \sqrt{Gm^3/\hbar^2 a}$, $R = \pi/k$ is the condensate radius and ρ_0 is the central number density of the condensate. $\phi(z,t)$ corresponds to the topological defect solution.

We are interested in characterizing the defect by its position z, hence we eliminate the Thomas-Fermi solution in GP equation, as well as its dependence on x and y coordinates, by multiplying (2) by ψ^*_{TF} and integrating it in these coordinates, obtaining

$$i\hbar\frac{\partial\phi(z,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} - \eta(z)\frac{\partial}{\partial z} + V + g(z)|\phi(z,t)|^2\right)\phi(z,t)\ , \tag{5}$$

where $\eta(z) = -\frac{\hbar k}{2m}\frac{\sin(kz)}{(1+\cos(kz))}$ is an extremely small factor and the term it couples to can be neglected.

The effective interaction parameter, $g(z) = g\rho_0 f(kz)/2$, is proportional to the central density and the form factor

$$f(x) = \frac{\ln\left(\frac{1}{x}\right) + \int_{2\pi x}^{2\pi} \frac{\cos(t)}{t} dt}{1+\cos(\pi x)}, \tag{6}$$

where $x = z/R$.

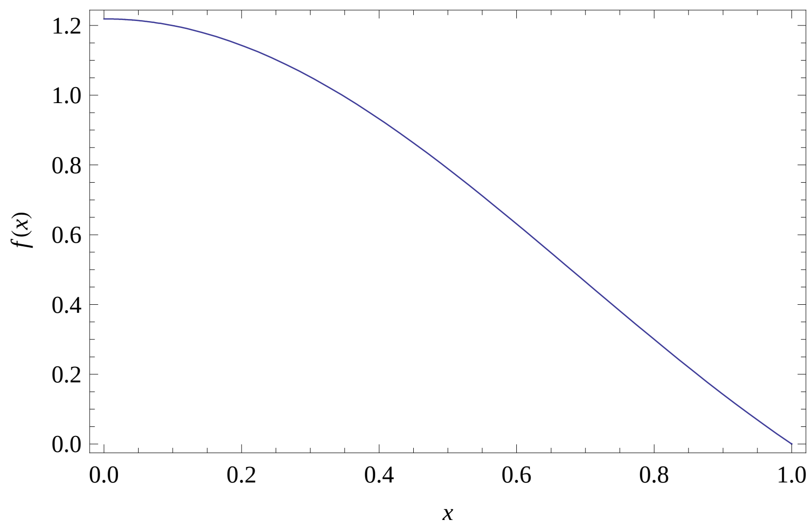

FIG. 2. Form factor of the condensate.

We assume that the width of the topological defect is much smaller than the size of the condensate. In this situation we can consider the particle number density and the effective interaction parameter as almost constants in the defect vicinity. As the self-graviting potential obeys the Poisson equation, we can approximate the potential as $V(\mathbf{r}) \approx V_0$ and, therefore, we substitute

$$\phi(z,t) = u(z,t)e^{-iV_0 t/\hbar} \tag{7}$$

in equation (5) to obtain the one-dimensional Gross-Pitaevskii equation for $u(z,t)$

$$i\hbar\frac{\partial u(z,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + g'|u(z,t)|^2\right)u(z,t), \tag{8}$$

where g' is supposed be locally constant. Equation (8) has the solution

$$u(z,t) = e^{-i\mu t/\hbar}\tanh\left(\frac{z-z_0}{\sqrt{2}\xi}\right), \tag{9}$$

where $\mu = g' = g(z_0)$. This solution is called a planar dark soliton, describing a domain wall at $z = z_0$, since the density vanishes at that point.

TABLE I. Values for the healing length ξ using masses and scattering lengths obtained in [1].

m (eV)	a (fm)	ξ (m)
10^{-6}	10^{-14}	10^{3}
10^{-5}	10^{-11}	10^{2}
10^{-4}	10^{-8}	10

The quantity ξ, called healing length, is related to the width of the domain wall. It is possible to show that it is a function of the parameters of the condensate in the form

$$\xi = \frac{1}{\sqrt{4\pi\rho_0 a f(kz)}}. \tag{10}$$

The density function for the domain wall $\rho_{DW} = |u(z,t)|^2$ is shown in figure 3.

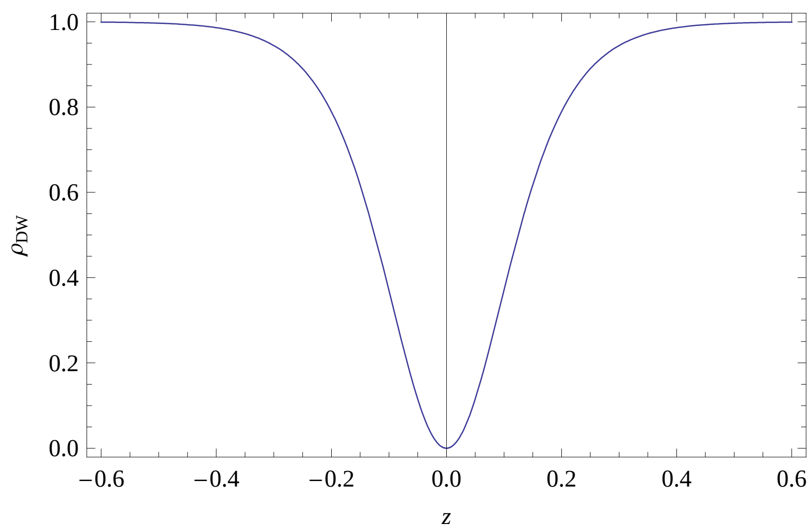

FIG. 3. Density function for the domain wall located at the origin of the coordinate system and with a healing length of 0.1 (in arbitrary units of length).

The density function

$$\rho_z = \int |\psi(\mathbf{r},t)|^2 dxdy \tag{11}$$

for the condensate with a domain wall is depicted in figure 4.

Local dark matter density measurements indicate a value of $0.4\,GeV/cm^3$. Using that information and the masses and scattering lengths for an axionic dark matter halo as suggested in [1], we can infer the order of magnitude for the healing length of the domain wall. The results are shown in table I.

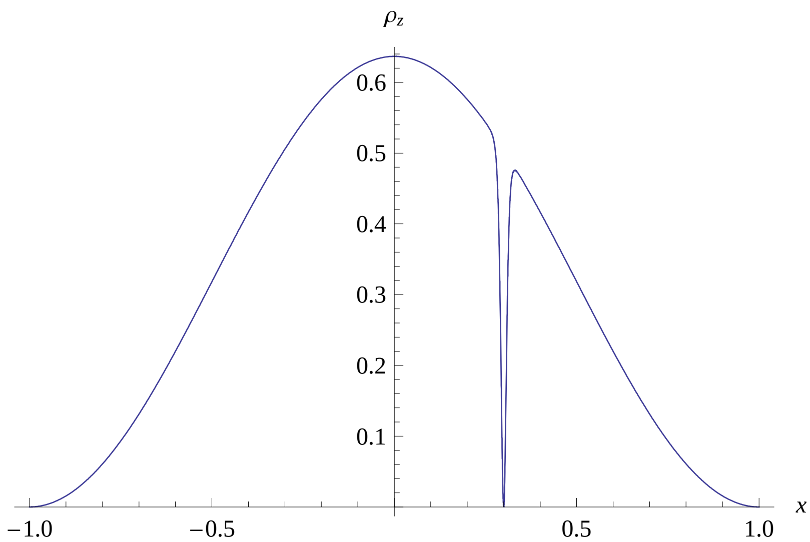

FIG. 4. Density function (in units of $k^2/(2\rho_0)$) for the condensate endowed with a domain wall. The width of the domain wall has been made large in order to facilitate visualization.

We can see that the healing length decreases very quickly with the particle's mass, becoming negligible for larger masses. The value for a mass of 10^{-4} eV is already beyond any physical significance at galactic scales.

III. GRAVITATIONAL EFFECT IN THE VICINITY OF THE DOMAIN WALL

We proceed now to the calculation of the effect of a domain wall located on the galaxy disk, more specifically crossing Earth's position, on a test body (e. g., a satellite).

In the presence of a domain wall, the total density distribution is not symmetrical, then the gravitational effects are distinct from the case of the halo density without the domain wall. We intent to estimate this difference by analysing the movement of a massive test body crossing the domain wall.

The gravitational field on the body is given by the solution of the equation

$$\nabla \cdot \vec{g} = -4\pi G \varrho, \tag{12}$$

where ϱ is the mass density which can be related to the wave function by

$$\varrho(x, y, z) = m|\psi(\mathbf{r}, t)|^2 \ . \tag{13}$$

The gravitational effect in the test body will be maximized if this body is moving along the z-axis and between the border and the center of the domain wall. By the symmetry of the spatial

configuration, the gravitational field has only a z-direction component. In this case, the equation to be solved is

$$\frac{\partial}{\partial z} g_z(z) = -4\pi G m\rho_0 \frac{\sin(kz)}{kz} \tanh^2\left(\frac{z - z_0}{\sqrt{2}\xi}\right). \tag{14}$$

The difference in the gravitational field is given by

$$g_z(z_0) - g_z(z_0 - \xi/2) \approx -4\pi G m\rho_0 \xi \frac{\sin(\pi x_0)}{\pi x_0} \left(\sqrt{2}\tanh\left(\frac{1}{2\sqrt{2}}\right) - 1\right), \tag{15}$$

where $x_0 = z_0/R$ is the domain wall position relative to the galaxy radius.

For the Sun's relative position, $x_0 \sim 0.5$ and the gravitational effect exerted in the massive body crossing the topological defect is of the order of 10^{-28} m/s^2, for a healing length of the order of 10^3 m and the gradient of the gravitational field is 10^{-31} $(m/s^2)/m$. Because the Earth's movement (along with the Sun) in the galaxy has a velocity of $\sim 10^5$ m/s, this effect in the vicinity of our planet could only be detected by an experiment with a precision greater than 10^{-32} m, which is far beyond present day technological capability.

IV. TANGENTIAL VELOCITY CORRECTION

When the domain wall is present, the gravitational field presents tangential components. However, the projection of the gravitational field in the tangential direction is much smaller than the projection in the radial direction even near the domain wall. Then, we can neglect the tangential components and assume that the gravitational field is radial and given by

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 g_r(r) = -4\pi G \varrho(r,\theta,\phi), \tag{16}$$

where

$$\varrho(r,\theta,\phi) = m\rho_0 \frac{\sin(kr)}{kr} \tanh^2\left(\frac{r\cos(\theta) - z_0}{\sqrt{2}\xi}\right). \tag{17}$$

Using Gauss theorem in the equation (16), we obtain

$$g_r(r) = -\frac{G M_{DM}(r)}{r^2}, \tag{18}$$

where the mass profile of the dark condensate galactic halo is,

$$M_{DM}(r) = \int_{\mathcal{V}} \varrho(r,\theta,\phi) d^3 r, \tag{19}$$

with $\mathcal{V}$ the volume of a sphere with radius r.

Equation (18) allows to represent the tangential velocity $v_{tg}^2(r) = rg_r(r)$ of a test particle moving in the halo as

$$v_{tg}^2(r) = v_{ss}^2(r) - v_{corr}^2(r) \ , \tag{20}$$

where

$$v_{ss}^2(r) = \frac{4\pi G m \rho_0}{k^2}\left(\frac{\sin(kr)}{kr} - \cos(kr)\right) \tag{21}$$

is the squared tangential velocity for the spherically symmetric case (already obtained in [7]) and

$$v_{corr}^2(r) = \frac{4\pi G m \rho_0}{k^2}\left(\sqrt{2}\pi\frac{\xi}{R}\Theta(r-z_0)\frac{\cos(kz_0)-\cos(kr)}{kr}\right) \tag{22}$$

is the correction in the squared velocity due to the presence of the domain wall. $\Theta(x)$ is the Heaviside step function.

v_{corr}^2 is proportional to $\xi/R \ll 1$, then the correction is maximal when the wall is located near the center of the galaxy. As the domain wall width is always many orders of magnitude smaller than the radius of the halo, this correction is also small.

In figure 5 both terms of (20) are shown, to stress the difference in magnitude they present.

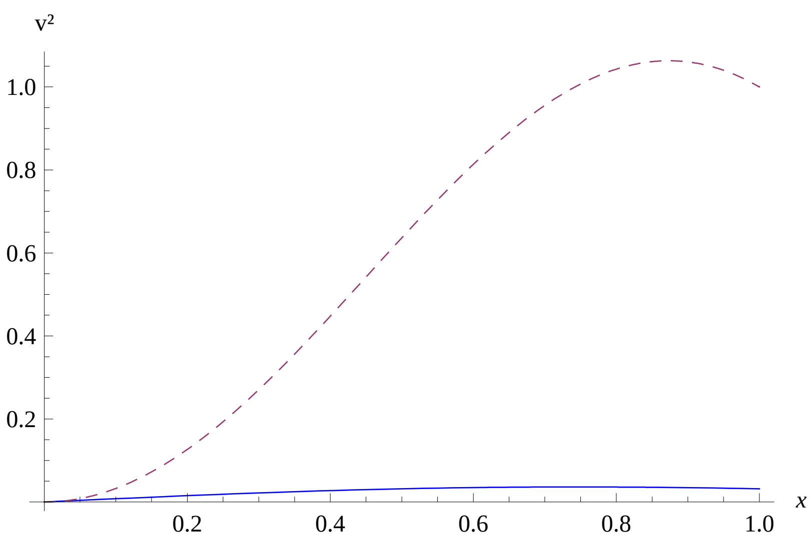

FIG. 5. Tangential velocities (in km^2/s^2) for the BEC dark matter halo (dashed line) and the domain wall (solid line). The wall's relative width ξ/R has been chosen as 0.05 to allow easy visualization of both curves. It is possible to see that the diference between the curves is very large. Here $x = r/R$.

With the addition of a barionic matter term (after choosing an appropriated barionic matter density profile), equation (20) can represent a rotation curve for stars in a spiral galaxy. The term

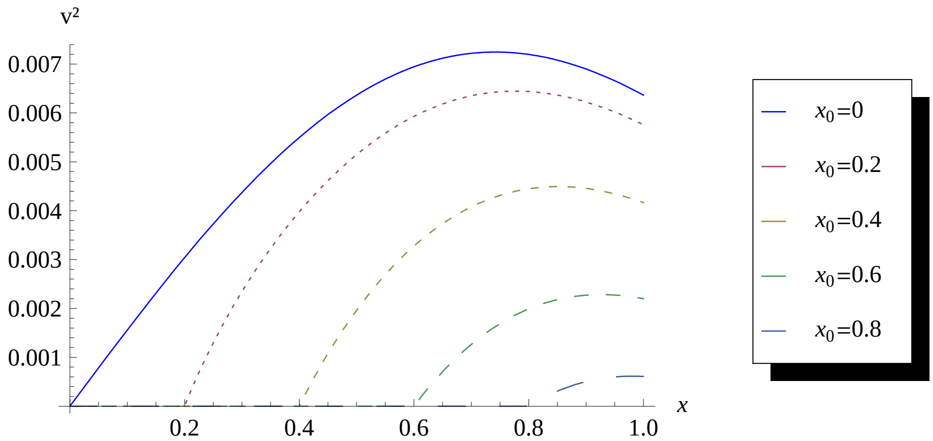

FIG. 6. Tangential velocity correction due to the influence of the domain wall with a relative width of 0.01. The curves are related to walls located in increasing relative distances x_0 from the center of the galaxy.

v_{ss}^2 had already been obtained in [7], and it was found to fit observed rotation curves for a number of galaxies. Our correction, v_{corr}^2, because of its small magnitude, cannot be detected in these types of rotation curves. As the factor ξ/R, for typical galaxies, would amount to about 10^{-16}, the ratio between the correction and the tangential velocity for the spherical symmetric case would be $v_{corr}/v_{ss} \sim 10^{-8}$.

The small efect that a domain wall would have in the galactic dynamics can be explained when we take in consideration the mass that could fill a disk the same size as the domain wall in a galaxy similar to the Milky Way ($R \approx 10\ kpc$). This mass would amount roughly to $10^{22}\ kg$, or about the mass of the Moon.

V. CONCLUSIONS

By assuming that the dark matter halo in galaxies is composed of a condensate of bosonic particles (with axionlike properties), as previous works hypothesized, we were able to model one type of substructure in the halo, in the form of a topological defect known as domain wall, derived from a dark soliton solution for the Gross-Pitaevskii equation in a self-graviting potential.

Because other types of topological defects (such as vortexes, monopoles, textures and ring solitons) would occupy a smaller volume in the halo, and therefore would have a smaller influence on the total dark matter density, we decided to restrict to the study of domain walls. Even in this case, the magnitude of the effects are too small to be subject to detection by present methods, at

least for the choice of parameters (mainly the healing length ξ, which depends on the mass and the scattering length of the axionlike dark matter particle estimated in [1]) and simplifications we have made here. For the local gravitational interaction, we have a gradient in the field of the order of 10^{-31} $(m/s^2)/m$. The correction factor on the velocity rotation curves for stars in spiral galaxies is typically of the order of 10^{-8}.

However, there may exist some kind of cumulative effect that renders the influence of domain walls considerable in a system with a larger number of topological defects or a greater dark matter density. They also may be important in other phases of galaxy evolution.

The main result of this work is the implementation of a methodology for the inclusion of topological defects in a quantum gas dark matter halo. The sequence of this study implies, for example, the determination of the dynamical and thermodynamical stability of the domain wall, the extension of the method for a spin-1 particle condensate (as suggested in [1]), and the manifestation of such defects in a fermionic quantum fluid, among other possibilities. These issues will be the subject of future work.

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